DATA FOR A UNIFIED THEORY OF TRANSPORT PROCESSES

Antal Laslo

Inzhenerno-Fizicheskii Zhurnal, Vol. 10, No. 1, pp. 60–63, 1966

UDC 532.50

The author has proved that Darcy's law is not dependent on the laws of Fick or Newton being a transport law that includes mechanical work.

Equations, describing transport processes involving heat, chemical components (diffusion) or momentum are well known. These laws were discovered independently of each other and have been designated Fourier's Law, Fick's law and Newton's law respectively, in honor of their discovers.

A unified interpretation of these laws has been established only in the past thirty years in the area of the thermodynamics of irreversible processes. This work may now be regarded as complete, and these terms have won universal acceptance in the international literature.

The situation regarding Darcy's law [1], also formulated in the mid-nineteenth century, is different. This law has not as yet been incorporated into the thermodynamics of irreversible processes. This situation is attributable to the lack of uniformity in references to Darcy's law, and also to the lack of clarification as to whether this law has evolved from those mentioned above.

This paper affirms that Darcy's law is not dependent on Fourier's, Fick's, or Newton's laws. An interpretation is outlined which can be incorporated in the modern system of the thermodynamics of irreversible processes.

It is important to bear in mind that Mokadam [2] studied this problem and succeeded in proving that Darcy's law is not dependent on the law of momentum transport. He emphasized that "Darcy's law cannot be derived from the Navier-Stokes equation, since this contains terms characterizing liquids only."

References to Darcy in the literature are not consistent or uniform. According to Le Goff [3] mass flowrate in granular material is proportional to the negative pressure gradient:

$$\rho v_x = - B \frac{\partial p}{\partial x};$$

$$\rho v \quad - B \operatorname{grad} p, \tag{1}$$

where the coefficient of proportionality B is permeability.

Le Goff also asserts that the extensive variable is mass, whereas the intensive variable characterizing the flow is represented by pressure. Therefore the mass flux density, a vector quantity, is introduced in the left side of equation (1).

Karman [4] stated Darcy's law in different forms. In one, the velocity of the moving medium is on the left side of the equation. This is worth mentioning since in this instance the unit of measurement of flow and the permeability dimension differ from those in equation (1). Permeability is, however, an important characteristic of granular material. According to Karman we have

$$v_x = - B_1 \frac{\partial p}{\partial x}. \tag{2}$$

Karman also expressed Darcy's law in terms of mass or component flux density as follows:

$$\frac{N_i}{At} = - B \frac{\partial p}{\partial x}. \tag{3}$$

These equations are given here since each of these forms (1), (2) or (3) is used in the international literature, with different symbols perhaps, but this is not important from the point of view of interpretation. The important point is that if interpretation (1) or (3) is correct, the assertion made by Darcy's law is analogous to that made by Flick's law. The only difference is in form. Thus, according to Fick the mass flux (in kilograms of moles) passing through unit area per unit time is proportional to the concentration gradient, whereas according to Darcy it is proportional to the pressure gradient. This is also Karman's interpretation [5].

From equations (1) and (3) the dimension of permeability is equal to B, kg/m · sec · atm(abs) or B, mole/m · sec · atm(abs), whereas from (2) B_1, m^2/sec · atm(abs).

The error in the interpretation of Darcy's law is explained by the fact that the external pressure on granular material is identical with the negative stress, known from mechanics, and is not proportional to the pressures of the individual components. For this reason the flux density of mechanical work L is expressed as an independent variable of state in the left side of Darcy's law, where ΔV—the difference in volumes—is the extensive quantity, and pressure (negative stress) is the intensive variable. Darcy's law should be properly stated in the following form:

$$j = L/At = \quad B^* \operatorname{grad} p.$$

In consequence, the unit of measurement for B* will be m^2/sec, which corresponds to the unified

Table 1
Transport Laws

Extensive variable	Notation and dimension	Density	Transport law	Diffusion coefficient	Convective flux density
Mass	m, kg	$\rho = \frac{m}{V}$	$j = -\mathscr{D}\ \mathrm{grad}\ \rho$	$\mathscr{D}$, m^2/sec	$j = \rho \bar{v}$
Component mass	$N_i = m_i/M_i$, moles	$c_i = \frac{N_i}{V}$	$j_i = -D\ \mathrm{grad}\ c_i$	D, m^2/sec	$j_i = c_i \bar{v}$
Heat	$Q = mc_p T$, kcal	$\rho c_p T = \frac{Q}{V}$	$j_q = -a\ \mathrm{grad}\ (\rho c_p T)$	$a = \lambda/\rho c_p$, m^2/sec	$j_q = \rho c_p T \bar{v}$
Momentum	$m\bar{v}$, kg · m · sec^{-1}	$\rho \bar{v} = \frac{m\bar{v}}{V}$	$j = -\nu\ \mathrm{grad}\ [\rho \bar{v}]$	$\nu = \eta/\rho$,	$j = \rho \bar{v} \bar{v}$
Mechanical work	$L = pV$, m^3 · atm(abs)	$p = \frac{pV}{V}$	$j = -B^*\ \mathrm{grad}\ p$	$B^* = B_1 p$, m^2/sec	$j = p \bar{v}$
Generalized expression	ψ	$\Gamma = \frac{\psi}{V}$	$j = -\delta\ \mathrm{grad}\ \Gamma$	δ, m^2/sec	$j = \Gamma \bar{v}$

Table 2
Relationship between Fluxes and Motive Forces

Flux	Motive force			
	concentration gradient	temperature gradient	pressure gradient	velocity gradient
Mass	Fick's law	Thermodiffusion effect	Pressure diffusion effect	No interaction
Heat	Diffusion thermo effect	Fourier's law	Heat-mechanical effect	"
Mechanical work	Osmotic concentration effect	Thermomechanical effect	Darcy's law	"
Momentum		No interaction		Newton's law

theory of transport processes (see Table 1, column 4).

In our opinion, what is new is that such a transport law, Darcy's law, corresponds both to the mechanical effect, known from the thermodynamics of irreversible processes, and to the other effects given in Table 2.

The assertion that fluxes should be interpreted in relation to mechanical work in the same way as in relation to mass, heat, or momentum, is formulated in Table 1, column 5. Generalized correlations are given in column 6.

It is easy to see that in this connection pressure is represented as density of mechanical work. The unit of dimension, introduced by the author for B* in Darcy's law, is $m^2 \cdot sec^{-1}$, surface area divided by time, which corresponds to the unified coefficients of diffusion, previously introduced for other values.

Thus, an incorrect understanding of permeability is eliminated from the literature and an interpretation of vital significance from the viewpoint of a unified and correct interpretation is advanced, i.e., that permeability is the coefficient of diffusion of mechanical work.

This interpretation of Darcy's law has not only theoretical, but also practical significance. Since Darcy's law does not follow from Fick's, a new, independent, dimensionless number, similar to those of Reynolds and Peclet, can be introduced. In the author's opinion, this should be called Darcy's number, if only to give him a deserved honor.

It is known that for momentum transport the relation between convective and diffusion flux is represented by the number Re of the medium

$$\mathrm{Re} = vd/\nu.$$

Similarly, values Pe and Pe', respectively, are introduced for the heat and chemical components:

$$\mathrm{Pe} = vd/a, \quad \mathrm{Pe}' = vd/D.$$

The introduction of the new dimensionless number proposed by the author is convenient, since it characterizes granular material (packing)

$$Da = vd/B^*.$$

In conclusion, it should be noted that the use of correlations employed for electric current becomes possible thanks to the unified interpretation, but this problem does not come within the scope of this paper.

NOTATION

a—heat diffusion coefficient, m^2/sec; A—surface area, m^2; B—permeability $mole \cdot m^{-1} \cdot sec^{-1} \cdot atm^{-1}$; B_1—permeability $m^2 \cdot sec^{-1} \cdot atm^{-1}$; B*—permeability $m^2 \cdot sec^{-1}$; c_i—component molar concentration, $mole/m^3$; c_p—specific heat, $kcal/kg \cdot deg$; d—characteristic length, m; D, $\mathscr{D}$—diffusion coefficients, m^2/sec; j—current density, m^2/sec; L—technical work, m^3/atm; m—mass, kg; M_i—molar weight of i-th component, kg/mole; N_i—no. of moles of i-th component; p—pressure, atm (abs); Q—quantity of heat, Kcal; T—temperature, deg; t—time, sec; v—linear flow velocity, m/sec; V—volume, m^3; δ—generalized diffusion coefficient, m^2/sec; λ—heat conductivity, $kcal/m \cdot sec \cdot deg$; ν—kinematic viscosity, m^2/sec; η—dynamic viscosity, $kg/m \cdot sec$; ρ—density, kg/m^3; Γ—generalized density, $1/m^3$; ψ-generalized value of extensive quantity.

REFERENCES

1. H. P. G. Darcy, Les fontaines publiques de la ville de Dijon. Paris, V. Dalmont, 1856.

2. R. G. Mokadam, Journal of Applied Mechanics, **28**, 208, 1961.

3. P. Le Goff, Benie chimique, 90, 161, 1963; Ind. Engng. Chem., **56**, 25, 1964.

4. P. C. Carman, Flow of Gases through porous Media. London, Butterworths Publ, p. 1, 1956.

5. Loc. cit., p. 3

28 April 1965

Institute of Chemical Engineering,
Vesprem, Hungary.